International Journal of Engineering, Science and Mathematics
Vol. 7Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com Double-Blind
Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Bianchi type-III Cosmic string coupled with Perfect fluid in Bimetric Relativity

V. Mahurpawar

Abstract

Five- dimensional Bianchi type-III cosmic string coupled with perfect fluid is considered in Rosen's bimetric theory of gravitation. It is observed that cosmic string couple with perfect fluid does not exist. Hence only vacuum model can be obtained.

Keywords:

Bianchi type-III;
cosmic string;
perfect fluid;
vacuum model;
bimetric relativity.

Author correspondence:

V. Mahurpawar
Associate Professor of Mathematics
Govt. Autonomous P.G. College, Chhindwara (M.P.), India

1. Introduction

Rosen [6] has modified the theory of gravitation by introducing a second metric tensor Υ_{ij} besides the Riemannian metric tensor g_{ij} at each point of space-time. Accordingly at each point of space-time one has two line elements-

$ds^2 = g_{ij}\,dx^i dx^j$. (1)

$d\sigma^2 = \Upsilon_{ij}\,dx^i dx^j$ (2)

The g_{ij} describe gravitation and interacts with matter. The back ground metric Υ_{ij} has no direct physical significance but appears in the field equations. Therefore it interacts with g_{ij} but not directly wit matter. One can regardΥ_{ij} as giving the geometric that would exist if there were no matter. ds is the interval between two neighboring events as measured by a measuring rod and a clock. The interval dσ is an abstract or geometric quantity not directly measurable.

Rosen has proposed the field equations by bimetric relativity as-

$K_i^j = N_i^j - \frac{1}{2} g_i^j = -8\pi\kappa T_i^j$ (3)

where$N_i^j = -\frac{1}{2}\gamma^{\alpha\beta}\left[g^{hi} g_{hj}\,|\alpha\right]|\beta$ (4)

Here vertical bar (|) denotes the γ- covariant differentiation.

$N = N_\alpha^\alpha, K = \left(\frac{g}{\gamma}\right)^{1/2}$ (5)

$g = det(g_{ij}), \gamma = det(\gamma_{ij})$ (6)

andT_i^j = Energy momentum tensor

equation (3) is obtained from Einstein field equation –

$G_i^j = R_i^j - \frac{1}{2} g_i^j R = 8\pi\kappa T_i^j$ (7)

Pursuing all the derivative of g_i^j of γ- differentiation. Using γ all derivatives does not change the physical contents of the field equations but it has some advantages. One can derive the Einstein field equations for empty space from the variation principle.Rosen [7] modified the version of bimetric relativity in accordance

with perfect cosmological principle.In this new version the back ground metric γ_{ij} is considered as a space-time of constant curvature which has the same degree of maximal symmetry as that of the flat space-time. It is very interesting to note that "Black hole" a creation of general theory of relativity does not exist in Bimetric theory of relativity.

Wang Xing et al [8], R. Bali [2] studied Bianchi type -III cosmological models with bulk viscosity in general relativity. Bianchi type- III string cosmological model with time dependent bulk viscous was studied by Raj, Bali [2] the behavior of model with presence and absence of bulk viscous was discussed. Adhav et al[1] were studied both meson afield and masonic perfect fluid models and found that only vacuum models can be obtained.Ronghe and Mahurpawar[5] ,Mahurpawar[4] were studied four dimensional Bianchi type-III models taking the energy momentum tensors cosmic sting and cosmic string coupled with perfect fluid and conclude that there is no contribution of cosmic string and cosmic string coupled with perfect fluid.

In this paper I have shown that higher five- dimensional cosmic string coupled with perfect fluid does not exist in Rosen's biometric theory of gravitation. Hence a vacuum model is obtained

2. Five- Dimensional Bianchi type –III Model and solutions-

We considered here the five dimensional space-time described by Bianchi type-III metric-

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2\alpha}dy^2 - C^2dz^2 - D^2du^2 \quad (8)$$

where A,B, C and D are functions of "t" and α is a constant.

The flat metric corresponding to equation (8) is

$$d\sigma^2 = dt^2 - dx^2 - dy^2 - dz^2 - du^2 \quad (9)$$

$$T_i^j = T_{istring}^j + (\epsilon + p)v_i v^j - \mathrm{p}g_i^h \quad (10)$$

$$T_{istring}^j = \rho u_i u^j - \lambda x_i x^j \quad (11)$$

Where$\rho$is the rest energy density of the system of string, λ is the tension density of the string, we consider

$$\rho = \rho_p + \lambda \quad (12)$$

Here$\rho_p$is the partial energy density attached to the string. The cosmological co-ordinate system is taken along x-direction i.e. $v_5 v^5 = 1$.u^irepresents the five- velocities and x^j represent an anisotropic direction i.e. direction of string.

We have $u_i u^j = -x_i x^j = 1$,$u_i x^j = 0$ (13)

εbeing the density p is the pressure . The components of energy momentum tensor for the cosmic string coupled with perfect fluid are-

$$T_i^j = -p + \lambda \ , T_2^2 = -p = T_3^3 = T_4^4 \text{ and } T_5^5 = \varepsilon + \rho \quad (14)$$

Rosen's Field equations (3) of biometric relativity for Bianchi type –III metric (8) with the help of (9) and equations (10) to(12) can be written as –

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(-p + \lambda) \quad (15)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = 16\pi\kappa p \quad (16)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = 16\pi\kappa p \quad (17)$$

$$+\left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 - \left(\frac{D_4}{D}\right)_4 = 16\pi\kappa p \quad (18)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 - \left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(\varepsilon + \lambda) \quad (19)$$

Using equations(15) to (19) we obtain

$$2\left(\frac{A_4}{A}\right)_4 = 16\pi\kappa p \quad (20)$$

$$2\left(\frac{A_4}{A}\right)_4 = -16\pi\kappa(-p + 2\lambda + \varepsilon) \quad (21)$$

Using equation (20) and (21) we get-

$$2\lambda + \varepsilon = 0 \quad (22)$$

By reality condition $\lambda \geq 0$ and$\varepsilon \geq 0$i.e.$\lambda = 0 = \varepsilon$ (23)

With the help of equation (23) the field equations (15) to (19) can be written as

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = 0 \quad (24)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = 0 \quad (25)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = 0 \quad (26)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 - \left(\frac{D_4}{D}\right)_4 = 0 \quad (27)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 - \left(\frac{D_4}{D}\right)_4 = 0 \quad (28)$$

By Equations (24) to (28)

$A = e^{c_1^t}$ (29)

$B = e^{c_2^t}$ (30)

$C = e^{c_3^t}$ (31)

$D = e^{c_4^t}$ (32)

Using equations (29) to (32)

The line element (8) becomes-

$ds^2 = dt^2 - e^{2c_1^t}dx^2 - e^{2c_2^t}e^{2\alpha}dy^2 - e^{2c_3^t}dz^2 - e^{2c_4^t}du^2$ (33)

By proper choice of coordinates this metric can be transform-

$ds^2 = d\tau^2 - e^{2\tau}[dx^2 + e^{2x}dy^2 + dz^2 + du^2]$ (34)

It is observed that the equation (34) is free from singularity at $\tau = 0$

3. Conclusion

We have studied Five-dimensional Bianchi Type-III anisotropic cosmological model taking cosmic string coupled with perfect fluid an energy-momentum in Rosen's Bimetric theory of gravitation and we observed that there no contribution of cosmic string coupled with perfect fluid to this model so Bianchi type-III model is free from singularity at τ=0, a vacuum model can be found.

References

[1] Adhav, K.S.; Khadekar, G.S.; Mate, V.G."Bianchi type-III cosmological model in four and five dimensional bimetric theory of relativity".Astrophy and space-Sc., Vol.295 Issue 3 ,pp 331-337,(2005).

[2] Bali, R. "Bianchi type-III inflationary cosmological model with builk viscosity", Pre space- time Journal Vol.5, No.10, pp981-986,(2014).

[3] Bali, R.; Pradhan, Anirudhha , "Bianchi type-III string cosmological models dependent bulk viscosity", arXiv:V:gr-qc/0611018.

[4] Mahurpawar,V,; Ronghe,A, "Cosmic string cloud cosmological model in bimetric theory of relativity", Acta. Ciencia.Indica Vol. XXXVII M No2 ,pp271-274, (2011).

[5] Ronghe,A. ; Mahurpawar, V., "Bianchi type- III cosmological model in bimetric relativity with cosmic string", Bull. Cal. Math. Soc. VOL.102, No. 4,pp315-318, (2011).

[6] Rosen, N. , "A bimetric theory of gravitation I" , Gel. Rel.Grav.Vol. 4, pp435- 447,(1973).

[7] Rosen,N., "Bimetric gravitation theory" , Gel. Rel. Grav.Vol. 9, No. 04. pp339- 351, (1978).

[8] Wang, X.X. , "Bianchi type- III string cosmology with bulk viscosity", Chin. Phys. Lett. 22, No 1 ,29, (JULY 2005).